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Reg. No. :

Code No. : 5093

Sub. Code : HMAE 43

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2020.

Fourth Semester

Mathematics

Elective – PARTIAL DIFFERENTIAL EQUATIONS

(For those who joined in July 2012-2015)

Time : Three hours

Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. A solution of $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$ is
 - (a) $xyz=c$
 - (b) $x^2=y$
 - (c) $x=y^2$
 - (d) $x=yz$
2. A pfaffian DE in Z variables is of the form
 - (a) $dx+P=0$
 - (b) $P+dy=0$
 - (c) $Pdx+Qdy=0$
 - (d) None

3. The *PDE* obtained by eliminating a, b from $z=(x+a)(y+b)$ is

(a) $px+qy=q^2$ (b) $p-q=z$

(c) $pq=z$ (d) $\frac{p}{q}=c$

4. Eliminating the arbitrary function f from $z=f(x-y)$ we got _____

(a) $px-qy=c$ (b) $p+q=0$

(c) $p-q=z$ (d) $p=q^2$

5. For the *PDE* $F(x,y,z,p,q)=0$, the equation $F(x,y,z,a,b)=0$ is a

(a) General integral

(b) Complete integral

(c) Particular integral

(d) Singular solution

6. $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k} \frac{\partial z}{\partial t}$ is

(a) Two dimensional equation

(b) One dimensional diffusion equation

(c) Wave equation

(d) Laplace equation

7. A complete integral of the equation $pq=1$ is

- (a) $ax+y=c$
- (b) $ax+y=z$
- (c) $a^2x+y-az=b$
- (d) $ax+by=c$

8. $Rr+Ss+Tt=f(x,y,z,p,q)$ is a

- (a) Legendre equation
- (b) pfaffian equation
- (c) Cauchy problem
- (d) Hyperbolic equation

9. $U_{xx}+U_{yy}=U_{zz}$ is

- (a) Parabolic
- (b) Hyperbolic
- (c) Elliptic
- (d) None

10. $\frac{\partial^2 z}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 z}{\partial x^2}$ is

- (a) One dimensional diffusion equation
- (b) Wave equation
- (c) Heat equation
- (d) Laplace equation

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Describe how will you solve $\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R}$.

Or

- (b) Find the integral curves of
$$\frac{adx}{(b-c)yz} = \frac{b dy}{(c-a)zx} = \frac{cdz}{(a-b)xy}.$$

12. (a) Find a *PDE* by eliminating arbitrary function from $F(u,v)=0$ where u,v are functions of x,y,z .

Or

- (b) Explain the method of solving the Lagrange's equation $P_p + Q_q = R$.

13. (a) Find the integral surface of the equation $(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z$ through the curve $xz=a^3, y=0$.

Or

- (b) Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2 + y^2 + z^2 = cxy$.

14. (a) Explain Laplace's equation.

Or

- (b) Find a particular integral of the equation $(D^2 - D^1)z = zy - x^2$.

15. (a) Describe Cauchy's problem for the second order *PDE*.

Or

- (b) By separating the variables, solve the *PDE*

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 z}{\partial t^2}.$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Show that a necessary and sufficient condition that the pfaffian differential equation $\bar{X}.dr=0$ should be integrable is that $\bar{X}.curl\bar{X}=0$.

Or

- (b) Solve $yz\,dx + 2xz\,dy - 3xy\,dz = 0$.

17. (a) Solve $px(x+y) = qy(x+y) - (x-y)(2x+2y+z)$.

Or

- (b) Find the integral surface of the *PDE* $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0$.

18. (a) Show that the equations $xp=yq$, $z(xp+yq)=2xy$ are compatible and solve them.

Or

- (b) Find the solution of the equation $Z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y)$ which passes through the x -axis.

19. (a) Explain charpit's method.

Or

- (b) Show that the characteristics of the equation $Rr+Ss+Tt=f(x,y,z,p,q)$ are invariant with respect to any transformations of the independent variables.

20. (a) Explain how you will solve hyperbolic equations of second order.

Or

- (b) Discuss the solutions of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{k} \frac{\partial z}{\partial t}$.
